

KOTEBE METROPOLITAN UNIVERSITY
COLLEGE OF NATURAL COMPUTATIONAL SCIENCES
DEPARTMENT OF MATHEMATICS
Course outline on Linear Algebra II (Math 2042)

Course objective and competency acquired

On the successful completion of this course students will be expected to

- # find the eigenvalues and eigenvectors of a square matrix,
- # identify similar matrices,
- # diagonalize a matrix when this is possible,
- # define inner product space,
- # find and apply the LU factorization of a matrix
- # understand the Gram-Schmidt process,
- # find an orthogonal basis for a subspace,
- # find an orthogonal complement of a subspace,
- # recognize and invert orthogonal matrices,
- # Comprehend the three canonical forms of matrices

Course description

This course covers the characteristic equation of a matrix, orthogonality, matrix factorizations, canonical forms, and direct sum decomposition of vector spaces, bilinear, quadratic and positive definite forms.

Chapter 1: The characteristic equation of a matrix (12 hrs)

- 1.1 Eigenvalues and eigenvectors
- 1.2 The characteristic polynomial
- 1.3 Similarity of matrices and characteristic polynomial
- 1.4 The spectral radius of a matrix
- 1.5 Diagonalization
- 1.6 Decomposable matrices
- 1.7 Minimal polynomial and Cayley-Hamilton theorem

Chapter 2: Orthogonality

- 2.1 The inner product
- 2.2. Inner product spaces
- 2.3. Orthonormal sets
- 2.4. The Gram-Schmidt orthogonalization process
- 2.5. Cauchy-Schwartz and triangular inequalities
- 2.6. The dual space
- 2.7. Adjoint of linear operators
- 2.8. Self-adjoint linear operators
- 2.9. Isometry
- 2.10. Normal operators and the spectral theorem
- 2.11. Factorization of a matrix (LU, Cholesky, QR) Singular value decomposition

Chapter 3. Canonical forms

- 3.1. Elementary row and column operations on matrices
- 3.2. Equivalence of matrices of polynomials
- 3.3. Smith canonical forms and invariant factors
- 3.4. Similarity of matrices and invariant factors
- 3.5. The rational canonical forms
- 3.6. Elementary divisors
- 3.7. The normal and Jordan canonical forms

Chapter 4. Bilinear and quadratic forms

- 4.1. Bilinear forms and matrices
- 4.2. Alternating bilinear forms
- 4.3. Symmetric bilinear forms and quadratic forms
- 4.4. Real symmetric bilinear forms

Chapter 5. Direct sum decomposition of vector spaces

- 5.1. Definition of a direct sum of vector spaces
- 5.2. Projection and invariant subspaces of a linear operator
- 5.3. Primary decomposition theorem

Assessment/evaluation

- ✓ Assignment (individual 10%, group 10%)
- ✓ Chapter quiz (15%)
- ✓ Test (20%)
- ✓ Attendance 5%
- ✓ Final exam 40%

Reference

- ➔ Serge Lang, Linear Algebra.
- ➔ Demissu Gameda, Topics in Linear Algebra.
- ➔ Schaum's Outline in Linear Algebra.