# Kotebe University of Education
# College of Science and Mathematics Education
# Department of Mathematics

## Course Syllabus on 'Advanced Topics in Combinatorics I'

**Course Code:** Math 745

**Credit Hours/ ECTS:** 3 / 7

**Lecture hrs:** 4

**Collaborative Learning hrs:** 2

**Independent learning hrs:** 5

**Length of time to complete the course:** 16 weeks

**Total number of load hours the student will carry to complete the course:** 189 hrs

**Pre-requisite course:** Math 642

**Program: M.SC. in Mathematics (Combinatorics Specialization)**

## Course Objectives:

Combinatorics is a branch of Mathematics that deals with the existence, enumeration, analysis, and optimization of discrete structures and states. Combinatorics has become an important branch of Mathematics since the middle of the twentieth century because of the major impact that computers have had, and continue to have, in our society. Large-scale problems that were previously difficult to solve are being solved with the advent of computers. But computers do not function independently; they need to be programmed to carry out these functions. The basis for these computer programs often are combinatorial algorithms for the solutions of problems. Analysis of these algorithms for efficiency and effectiveness with regard to running time and storage requirements requires clever combinatorial thinking and enumeration. In this course, students will be introduced to fundamental techniques in the field ranging from basic principles of counting, whose simplicity masks both the power and the ease with which they can be misused, to more advanced techniques, such as Lagrange Inversion Formula. One of the unique features of this course is that students will be introduced to the Wilf-Zeilberger method and Zeilberger's algorithm to obtain and prove combinatorial identities involving proper hyper-geometric functions with the help of computer-generated recurrence.

# Learning Outcomes

On completion of the course successful students will be able to:

- demonstrate ability to think critically and analytically by using appropriate enumeration techniques to solve problems from diverse areas.
- formulate and solve problems from diverse areas using application of specific analysis and/or generating function models.
- work effectively in groups on projects that require an understanding and use of combinatorial analysis and provide answers to enumeration problems with non-trivial constraints in clear, coherent, and convincing manner.
- locate and use information on combinatorics and its applications.

Mode of Delivery: This course will be offered in a semester based mode of delivery

# Course Contents

1. **Review of Generating Functions**
   - 1.1 Ordinary and Exponential Generating Functions
   - 1.2 Lagrange Inversion Formula and its Application
   - 1.3 Recurrence Relations and the Snake-oil Method
   - 1.4 Stirling Numbers of the Second Kind and Bell Numbers
2. **Enumeration of Combinatorial Objects**
   - 2.1 Counting Trees and Forests
   - 2.2 Acyclic and Parking Functions
   - 2.3 Counting Polyominoes
   - 2.4 Counting k-colored graphs
   - 2.5 Counting Magic Squares and Cubes
   - 2.6 Counting Latin Squares and Hall's Theorem
3. **Combinatorial Identities**
   - 3.1 Bijective Counting and Combinatorial Identities
   - 3.2 Riordan Groups
   - 3.3 Proving Combinatorial Identities using Riordan Arrays
   - 3.4 The Fundamental Theorem of WZ Theory
   - 3.5 WZ Pairs and Combinatorial Identities

4. **Counting Permutations**

    4.1 Eulerian Numbers

    4.2 The Cycle Structure of Permutations

    4.3 Stirling Numbers of the First Kind

    4.4 Permuatations of a given Kind

    4.5 Inversions and Counting Permutations with respect to Inversions

5. **(Optional) Combinatorial Sequences**

    5.1 Unimodality

    5.2 Log Concavity

    5.3 The Real Zeros Property

# Teaching-Learning Strategy/Methods

Lectures, Tutorial, Group Assignments

# Assessment Strategy/Methods

- Assignment: 20%
- Project and Test: 30%
- Semester Examination: 50%

# Course Policy

A student has to

- Attend at least 80% of the classes
- Take all continuous assessments
- Take final examination
- Respect all rules and regulations of the University Reference

# References


[1] Aigner, M., A course in Enumeration, Springer, 2000

[2] Bona, M., A Walk Through Combinatorics, 1st Edition, World Science Publishing Company, River Ridge, New Jersey, 2002.

[3] Graham, R.L., Kunth, D.E., and Patashnik, O., Concrete Mathematics, Addison-Wesley Publishing Company, 1989.

[4] Lando, K., Lectures on Generating Functions, American Mathematical Society-Student Mathematical Library Series, Providence, Rhode Island, 2003.

[5] Pemmaraju, S., Computational Discrete Mathematics Combinatorics and Graph Theory with Mathematica, Cambridge University Press, Cambridge, 2000.

[6] Stanley, R. P. Enumerative Combinatorics, Vol I, Monterey: Worseworth & Brooks / Cole, 1986.

[7] Stanley, R. P. Enumerative Combinatorics, Vol II, Cambridge University Press, 1999.

[8] Wilf, H., Generating functionology, 2nd edition, Academic press, San Diego, CA 1994

[9] Van Lint, J.H. and Wilson, R.M., A course in Combinatorics, Cambridge University Press, Cambridge, 1992